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恐龙蛋壳的生物力学性质(VI)
—— 在外力作用下恐龙蛋壳结构的稳定性

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摘要 应用薄壳理论分析五种类型恐龙蛋壳的受力特性，求出它们在不同状态下埋在沙土中的失稳临界载荷。结果表明，不同类型恐龙蛋在蛋窝中的不同排列方式是与其蛋壳的抗失稳能力的大小密切相关，是某些类群的恐龙在产卵时为解决其低强度蛋壳在保护卵不受外力损伤和在卵的孵化后期幼雏能够破壳而出这两方面的矛盾而采取的一种保护性措施。

关键词 恐龙蛋，薄壳，临界压力，临界应力，失稳，破碎强度

中图法分类号 Q915.21，Q66

一、前 言

不同类型的恐龙蛋在蛋窝中排列的方式各不相同(杨钟健，1965；赵资奎，1975，1979；赵资奎，李荣，1993)。某些类型，如长形蛋科的各个类群和棱齿龙蛋等，它们并不是随意放置或自然地平躺在蛋窝中，而是很有规律地排列着，蛋的长轴与地面成一定的角度(图版I)。然而，有的类群的蛋，如椭圆形蛋等在蛋窝中则没有一定的排列规律。

赵资奎等(1994)提出，不同类型的恐龙蛋，虽然其形状大小各不相同，但大体上为旋转对称外形。由于这些蛋的蛋壳厚度远小于轴的长度，也远小于转动半径，从力学的角度上，可以把它看成是一个绕其长轴旋转而成的壳体。当它们被产下埋在沙土中孵化时，就会受到分布压力的作用。如果当此压力达到临界值 p_{cr} 时，在蛋壳的某一部位上将突然出现凹陷，壳也随之而破坏。此压力值 p_{cr}，称为壳的失稳临界载荷。研究表明，在外力作用下不同类型的恐龙蛋壳具有不同的失稳临界载荷，也就是说，不同类型的恐龙蛋壳有不同的抗失稳能力(马和中，赵资奎，1994)。这就提示我们，不同类型的恐龙蛋在蛋窝中有不同的排列形式可能与其蛋壳本身的抗失稳能力的大小有关。

因此，要了解不同类型恐龙蛋各自在蛋窝中不同排列方式的古生物学意义，就必须先假设每一种类型的恐龙蛋以不同的放置形式埋在沙土中，在这种情况下研究其受力特

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征，找出其处于不同状态下受压破损的抵抗能力。

本文以五种恐龙蛋化石作为研究材料(参看赵资奎等，1994；马和中，赵资奎，1994)，它们的几何形状数据如表1所示，表中蛋型组别E为棱齿龙蛋的代表 *Prismatoolithus gebiensis*（赵资奎，李荣，1993)。

表1　五种恐龙蛋的几何数据(厘米)

Table 1　Geometric data of five types of dinosaur eggs (cm)

蛋型组别 Egg type	A	B	C	D	E
蛋尖端球径 Diameter at the pointed end	2.3	3.8	3.4	3.0	2.0
蛋钝端球径 Diameter at the blunt end	2.6	4.4	4.5	3.8	2.5
蛋的长轴 Long axis of the egg	9.4	20.0	18.0	14.5	12.0
蛋壳厚度 Eggshell thickness	0.24— 0.26	0.14— 0.18	0.14— 0.17	0.12— 0.14	0.07— 0.09

二、恐龙蛋平放埋于沙土中的受力分析

如果恐龙蛋是平放地埋在沙土中，此时蛋的长轴与地面平行。在这种情况下，它可能出现两种破坏形式：一种是壳体受压缩而破裂；另一种是因压力而失稳(即向内凹陷屈曲)。如果蛋内为全部不可压缩的液体填满，则第二种破坏形式发生时要受到阻力。但由于蛋内有气室，故在气室体积的小变化下(内压力增大较小)，就可能造成蛋壳的内陷屈曲而折裂。根据固体力学的理论分析与实验，结果表明不论是压碎破坏或失稳内陷屈曲，一般多发生在蛋的中部(赵资奎等，1994)。这一部位可以近似地看成是受分布外压力作用的锥形薄壳，在外压达到某一确定值时即出现向内的凹陷(屈曲)。产生凹陷的分布压力称为临界分布压力 $p_{\Phi\mathrm{cr}}$. 试验表明，$p_{\Phi\mathrm{cr}}$ 的大小主要与蛋壳几何参数(径、厚度、长度和锥角)及材料性质(弹性模量、泊松比)有关，其它因素的影响较小。由固体力学的壳体稳定性研究(Hyman & Healey，1967)得到用于计算薄壳的临界分布压力公式为：

$$p_{\varphi\,\mathrm{cr}}=\frac{K_{\varphi}E}{1-\mu^{2}}\left(\frac{h}{\overline{R}}\right)^{2}\text{。} \qquad (1)$$

式中 h 为壳厚度，$\overline{R}$ 为折算半径，可由下式

$$\overline{R}=(R_{\max}+R_{\min})\;/\,2\cos\gamma \qquad (2)$$

得到。式中 R_{max} 及 R_{min} 分别为锥体两端的半径，γ 为锥角。(1)式中 E 为材料的弹性模量，μ 为材料的泊松比。K_φ 为外压临界载荷系数，可由图 1 查得。图中 b 为壳的长轴。

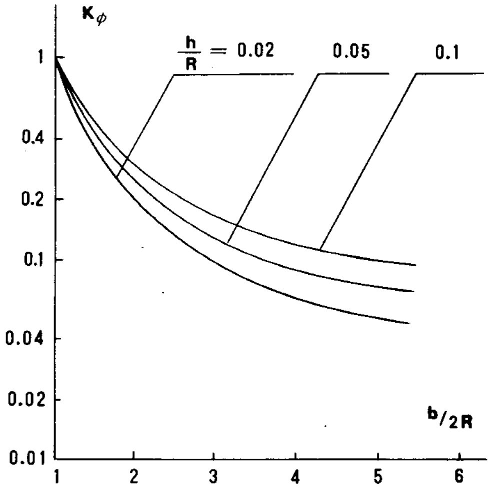

图 1 外压临界载荷系数 K_φ 变化曲线
Fig.1 Variations in K_φ

在求得 $p_{\varphi cr}$ 后，可根据壳的薄膜理论求得出现凹陷时壳中的临界应力：

$$\sigma_{\varphi cr}=p_{\varphi cr}\overline{R}/h \tag{3}$$

由于锥角 γ 很小，可近似取 $\cos\gamma=1$（此处误差在 5% 以下）。恐龙蛋壳已成为化石，它在新鲜情况下的弹性模量 E 和破坏应力 σ_b 均不知道。然而，恐龙蛋壳和鸟蛋壳一样，主要由方解石微晶及少量有机基质组成，尤其是本文研究的这 5 种化石标本，其蛋壳的基本结构单元和排列形式一般均与鸟蛋壳的基本相似。因此可以参考鸟蛋壳的情况来研究。

鸟蛋壳材料的 μ 一般为 0.25，故本文研究的恐龙蛋壳材料 $\mu=0.25$。有关 5 种恐龙蛋的抗压能力 $p_{\varphi cr}$ 及 $\sigma_{\varphi cr}$ 等数据如表 2 所示。

表 2 不同类型恐龙蛋的临界外压 $p_{\varphi cr}$ 及临界应力 $\sigma_{\varphi cr}$ 比较

Table 2 Representative parameters of different types of dinosaur eggshells

蛋形组别 Egg type	A	B	C	D	E
长轴 L Long axis (cm)	9.4	20.0	18.0	14.5	12.0
半径 $\overline{R}$ Conversion radius (cm)	2.95	4.10	3.96	3.40	2.25
壳厚 h Eggshell thickness (cm)	0.24	0.14	0.14	0.12	0.07
$h/\overline{R}$	0.081	0.034	0.035	0.035	0.031
$L/2\overline{R}$	1.59	2.44	2.28	2.13	2.67
K_φ	0.46	0.17	0.175	0.19	0.15
$(p_{\varphi cr}/E)\times10^4$	32.5	2.11	2.33	2.52	1.55
$(\sigma_{\varphi cr}/E)\times10^4$	399.0	61.9	66.0	71.6	49.8

根据对鸟蛋壳的统计，可知$\sigma_b=0.025E$。那么从表2可以看出，A型恐龙蛋的屈曲应力$\sigma_{\varphi crA}=399E/10^4=0.0399E>\sigma_b$，这就是说，这一类型的蛋壳首先是其应力要达到$\sigma_b$才发生破裂。然而，对于B, C, D, E四种类型的恐龙蛋来说，情况正好相反，以D型恐龙蛋为例：$\sigma_{\varphi crD}=71.6E/10^4=0.00712E<\sigma_b$，很明显，这四种类型恐龙蛋壳，只要其应力达到$\sigma_{\varphi cr}$这一屈曲临界应力便发生破裂。因此，如果B, C, D, E四种恐龙蛋是平放地埋在沙土中就可能在很小载荷下因屈曲而破坏。

三、恐龙蛋在竖立情况下埋在沙土中的受力分析

如果把本文研究的五种恐龙蛋竖立起来埋在沙土中来研究其受力性质，那么，可把蛋的两端看作是承受分布外压作用的圆球壳，把蛋的中部看作是支持两端受轴压作用的圆锥壳。现分别计算其临界压力。

1. 关于蛋的两端受外压失稳的临界压力计算

作为受均匀分布外压的薄圆球壳，由固体力学理论得到的失稳(屈曲)临界外压强p_{xcrs}的计算公式为：

$$p_{xcrs}=K_1K_2K_3E(h/R)^2\ , \tag{4}$$

式中h为壳厚度，R为球半径，K_1为理论推导中得到的计算系数(Timoshenko & Gere, 1961)：

$$K_1=2/\sqrt{3(1-\mu^2)}\ 。\tag{5}$$

K_2是考虑在实际情况下蛋壳不一定是理想球体，厚度也不可能均匀而引入的形状偏离系数，它随h/R的减小而减小，可由图2查得。K_3是考虑在载荷不均匀或在受力前即有凹陷情况下而引入的初始缺陷系数，当载荷增加时可由于凹陷的扩大而加速失稳。K_3可由图3查出。

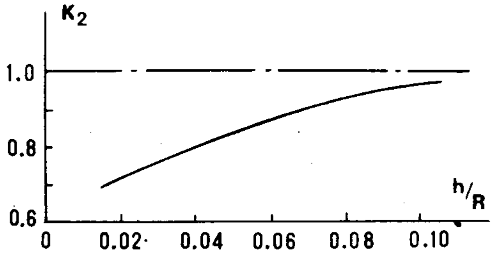

图2　形状偏离系数K_2随h/R变化曲线
Fig.2　Variations in K_2 with the h/R

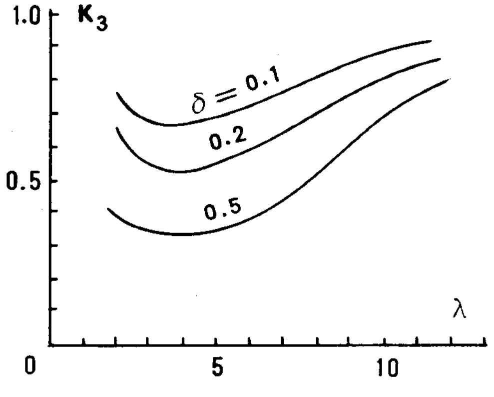

图3　缺陷系数K_3随λ及δ的变化曲线
Fig.3　Variations in K_3 with λ and δ

图3中的横坐标：

$$\lambda=[12(1-\mu^2)]^{1/4}(R/h)^{1/2}\ , \tag{6}$$

δ 代表壳的凹陷度

$$\delta = \Delta / h, \tag{7}$$

式中 Δ 为实际上最大凹陷量。

由公式(4)可知，在蛋的钝端，h/R 值较小，所以 p_{xcrs} 值也小，也就是说其抗外压能力低，比蛋的尖端更易失稳。故本文只研究钝端 p_{xcrs} 及 σ_{xcrs} 值，其计算结果见表 3。取 $\mu=0.25$，$\delta=0.2$。根据下式可求出临界应力

$$\sigma_{\mathrm{xcrs}} = p_{\mathrm{xcrs}} R / (2h)。 \tag{8}$$

表 3 恐龙蛋钝端蛋壳临界外压 p_{xcrs} 及临界应力 σ_{xcrs}

Table 3 Representative parameters at the blunt end of different types of dinosaur eggshells

蛋形组别 Egg type	A	B	C	D	E
半径 $\overline{R}$ Conversion radius (cm)	3.1	4.4	4.5	3.8	2.5
壳厚 h Eggshell thickness (cm)	0.26	0.18	0.17	0.14	0.09
h/R	0.084	0.041	0.038	0.037	0.036
K_2	0.96	0.82	0.79	0.78	0.76
λ	6.32	9.05	9.42	9.54	9.65
K_3	0.61	0.77	0.80	0.81	0.82
$(p_{\mathrm{xcrs}}/E)\times 10^4$	49.1	12.6	10.8	10.2	9.63
$(\sigma_{\mathrm{xcrs}}/E)\times 10^4$	586	308	285	278	267

2. 关于蛋的中部受轴压失稳临界压力与临界应力计算

正如上面提到，可把蛋的中部看作是支持蛋两端受轴压作用的圆锥壳，其失稳临界总载荷(Weingarten *et al.*, 1965) 为:

$$P_{\mathrm{xcr}} = 2\pi K_{\mathrm{x}} h^2 \cos^2\gamma\, E\,, \tag{9}$$

式中:

$$K_{\mathrm{x}} = \frac{1}{\sqrt{3(1-\mu^2)}} = 0.546\left[1 - exp\left(-\frac{1}{16}\sqrt{\frac{\overline{R}}{h}}\right)\right] + 0.9\left(\frac{\overline{R}}{L}\right)^2\left(\frac{h}{R}\right)。 \tag{10}$$

相应的外力为:

$$p_{\mathrm{xcr}} = 2K_{\mathrm{x}}(h/\overline{R})^2\cos^2\gamma E\,, \tag{11}$$

应力为:

$$\sigma_{xcr}=P_{xcr}/(2\pi\overline{R}h)=K_x h E\cos^2\gamma/\overline{R}. \tag{12}$$

以上各数值计算结果见表 4。

表 4　恐龙蛋中部锥体壳轴压 P_{xcr}，p_{xcr} 及 σ_{xcr}

Table 4　Representative parameters of the egg′s middle portion of five types of dinosaur eggshells

蛋形组别 Egg type	A	B	C	D	E
半径 $\overline{R}$ Conversion radius (cm)	2.95	4.10	3.96	3.40	2.25
壳厚 h Egghsell thickness (cm)	0.24	0.14	0.14	0.12	0.07
K_x	0.489	0.440	0.442	0.442	0.433
$(P_{xcr}/E)\times10^2$	17.1	5.42	5.44	4.00	1.33
$(p_{xcr}/E)\times10^4$	64.7	10.3	11.0	11.0	8.4
$(\sigma_{xcr}/E)\times10^4$	398	150	156	156	135

从表 2，表 3 及表 4 中列出的不同类型恐龙蛋的 p_{xcr} 及 σ_{xcr} 等数据的比较可以看出，B, C, D, E 等四种类型恐龙蛋的抗失稳能力很低。如果把它们竖立起来埋在沙土中，则比平放地埋在沙土中更能承受较大外压。

四、恐龙蛋斜放在沙土中的受力分析

如果把恐龙蛋斜放在沙土中，这时蛋的长轴与地面成β角，其轴向分布压力

$$p_A=p\sin\beta\ , \tag{13}$$

侧向分布压力

$$p_B=p\cos\beta\ 。 \tag{14}$$

根据壳体薄膜理论，p_A 产生的轴向应力

$$\sigma_x=p_A R/(2h)=pR\sin\beta/(2h), \tag{15}$$

p_B 产生的周向应力

$$\sigma_\Phi=p_B\overline{R}/h=p\overline{R}\cos\beta/h。 \tag{16}$$

在 p_A 作用下轴向能承受的最大(临界)应力

$$\sigma_{xcr}=p_{xcr}\overline{R}/(2h)=K_x hE\cos^2\gamma/\overline{R}, \tag{17}$$

在 p_B 作用下周向能承受的最大(临界)应力

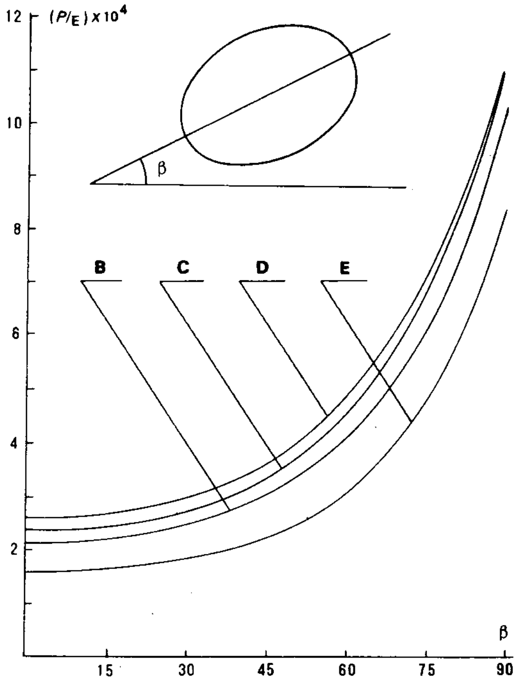

图4 B，C、D 和 E 四种类型恐龙蛋的 p_{cr}/E 随 β 变化曲线

Fig.4 Variations in the critical pressure with the angle β in four types of dinosaur eggshells

$$\sigma_{\varphi cr}=\frac{KE}{1-\mu^2}\left(\frac{h}{\overline{R}}\right)\cos\gamma。\quad (18)$$

根据壳体理论，在有双向应力的情况下，当

$$(\sigma_x/\sigma_{xcr})^2+(\sigma_\varphi/\sigma_{\varphi cr})^2=1\quad (19)$$

时，就会发生壳的失稳破坏。将(15)—(18)四式代入(19)式，取$\cos\gamma=1$，可得到

$$\frac{P_{cr}}{E}=\frac{1}{(G\sin^2\beta+H\cos^2\beta)^{1/2}},\quad (20)$$

其中

$$G=\frac{1}{4K_X^2}\left(\frac{\overline{R}}{h}\right)^4,$$

$$H=\frac{(1-\mu^2)^2}{K^2}\left(\frac{\overline{R}}{h}\right)^4\quad (21)$$

根据(20)和(21)式可以计算出不同类型恐龙蛋以不同的 β 角度埋在沙土中的临界应力。计算结果列于表 5，并在图 4 中绘出曲线。

表 5 恐龙蛋以不同倾角 β 埋在沙土中的抗外压能力

Table 5 Values of break pressure caculated from different inclination of five types of dinosaur eggs

蛋形组别 Egg type		A	B	C	D	E
半径 $\overline{R}$ Conversion radius (cm)		2.95	4.10	3.96	3.40	2.25
壳厚 h Eggshell thickness (cm)		0.24	0.14	0.14	0.12	0.07
K_x		0.489	0.440	0.442	0.442	0.433
K_φ		0.46	0.17	0.175	0.19	0.15
$G/10^4$		2.39	95.0	81.9	82.5	142.3
$H/10^4$		9.49	2237	1837	1569	4170
$\frac{p_{cr}}{E}\times10^4$	0°	32.5	2.11	2.33	2.52	1.55
	15°	33.3	2.19	2.41	2.61	1.60
	30°	36.0	2.42	2.67	2.89	1.78
	45°	41.1	2.93	3.23	3.48	2.15
	60°	49.0	3.98	4.38	4.69	2.95
	75°	59.1	6.48	7.08	7.40	4.93
	90°	64.7	10.30	11.00	11.00	8.40

五、讨　论

在本文研究的五种类型的恐龙蛋中，A型恐龙蛋壳，如 *Ovaloolithus* 具有很高的强度，不论它们埋在沙土中是处于横躺位置或者竖立位置，其失稳临界压力 σ_{cr} 均高于压碎破坏应力 σ_b 。因此，以这类蛋为代表的恐龙在筑巢产卵时，不论它把卵以何种角度产在蛋窝中，在一般情况下均不易受压破裂。从已发现的这一类型的蛋化石来看，它们在蛋窝中均呈不规则的排列方式(杨钟健，1965)，这种情况恰好与本文的研究结果一致。这就进一步说明，由 *Ovaloolithus* 为代表的恐龙可把卵随机产在蛋窝中。

然而，B、C、D 和 E 四种类型，也就是说如 *Macroolithus*、*Elongatoolithus*、*Nanhsiungoolithus* 及 *Prismatoolithus* 等的蛋壳，其强度则比较低。如果把它们平放地埋在沙土中，其承载能力也很低。当其临界应力 σ_{cr} 达到压碎破坏应力 σ_b 的 1/4 至 1/6 时，在该蛋的中部就可能因受外压失稳而破坏。如果这些蛋是倾斜地(蛋的长轴与地平面呈一定的角度)埋在沙土中，则蛋的中部抗失稳能力有所提高。当 β 角在 45° 以下时，这种提高并不明显。当 β 角达到 60° 时，失稳临界压力 p_{cr} 可以提高到接近平放时的2倍。在 $\beta=75°$ 时，则可达到平放时的3倍。如果把蛋竖立起来(即 $\beta=90°$)埋在沙土中，它的 p_{cr} 可达到平放时的4—5倍。这时蛋的两端和中部的抗失稳能力大致相等，它们所对应的临界应力 σ_{cr} 均接近于蛋壳材料的压碎应力 σ_b。因此对于强度较差的恐龙蛋类型来说，只要使它们的长轴与地面成一定的角度埋在沙土中，就可以在很大程度上改善这些蛋壳的强度，降低它们受压破坏的危险程度。

棱齿龙产卵时，是把蛋一个个竖立或倾斜地埋在沙土中(Horner，1984；赵资奎，李荣，1993)。从上述的分析结果来看，以E型蛋为代表的棱齿龙蛋在 70°—90° 的角度埋在沙土中可以达到最大抗压效果。如果以此为标准，那么以长形蛋科为代表的B，C，D三种恐龙蛋，只要以 45°—75° 的倾斜放置在蛋窝中，就可以达到棱齿龙蛋处于 75°—90° 倾斜放置时的抗失稳能力。

上述这一分析结果与已发现的由长形蛋科为代表的B，C，D型在蛋窝中有固定的排列形式相一致(甄朔南，王存义，1963；杨钟健，1965；赵资奎，1975)。我们有理由认为，由于以长形蛋科为代表的B，C，D型蛋和棱齿龙蛋(E型蛋)的蛋壳强度较低，不能有效地起保护作用。解决这一问题的有效方法之一就是使蛋与地面成一定的角度，以提高蛋壳的抗外载荷能力。因此，为了保证卵在孵化期间内能有效地防止蛋受压破损，以这些蛋为代表的恐龙必须从生殖行为上来改善它们所产的卵的存放状态，以克服蛋壳结构本身的弱点。所以它们在产卵时，把卵有规律地与地面成一定的角度排列在蛋窝中是非常必要的和合理的。这就充分表明，恐龙的智力可能比人们长期以来想象的要高。

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BIOMECHANICAL PROPERTIES OF DINOSAUR EGGSHELLS(VI) —— THE STABILITY OF DINOSAUR EGGSHELL UNDER EXTERNAL PRESSURE

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Summary

It is well known that the patterns of dinosaur egg arrangement within the clutch differ from group to group (Young, 1965; Zhao, 1975, 1979; Zhao and Li, 1993). Some types such as elongatoolithid and hypsilophodontid eggs were laid regularly in nest. The long axis of these eggs forms certain angle with the ground (Plate I). But as for others, they were disorderly arranged in the nest.

Zhao *et al.* (1994) advanced that dinosaur eggshells could be considered as rotational thin shell. When the dinosaur eggs were laid, and buried in sand for

incubation, they were subjected to distributive pressure. Once the pressure comes to the critical value p_{cr}, the eggshell will subside and then break. Here p_{cr} is called the critical pressure of instability. It has been demonstrated that Variation between p_{cr} of different kinds of dinosaur eggshells existed (Ma and Zhao, 1994). This suggests that arrangement patterns of dinosaur eggs in nests might have something to do with the eggshell's ability to resist external pressure.

The purpose of this paper is to discuss the relationship between the pattern of egg arrangement in the nest and the critical pressure of its eggshell, and five types of dinosaur eggshells are available.

1. Analysis of mechanical properties of dinosaur eggs buried evenly in sand

If dinosaur eggs were buried evenly in the ground, two breaking patterns would appear under the external pressure. One is that eggshells were broken due to being compressed. The other is eggs turned instable because of external pressure (i. e. subsidence appeared on the eggshell surface). If eggs were filled with incompressible liquid, the second breaking pattern was not easy to happen. But because the egg contains air cell, a slight variation in volume of air cell would result in subsidence and breakage on eggshell. Theoretical analysis and experiments showed that no matter which breaking pattern happened eggs were most often broken in the middle portion (Zhao *et al.*, 1994). This portion may proximally be regarded as conical thin shell under distributive external pressure. It has been demonstrated from bird eggshells that the critical distributive pressure $p_{\varphi cr}$, is mainly determined by the geometric data of eggshells (radius, thickness, length and conical angle) and the nature of eggshell materials (elastic modulus, Poisson's ratio). $P_{\varphi cr}$ can be calculated by the following formula:

$$p_{\varphi cr}=\frac{K_\varphi E}{1-\mu^2}\left(\frac{h}{\overline{R}}\right)^2. \tag{1}$$

Where h is the shell thickness, K_Φ is the critical load coefficient of external pressure and can be obtained from figure 1, $\overline{R}$ is the conversion radius and:

$$\overline{R}=(R_{\max}+R_{\min})\ /(2\cos\gamma), \tag{2}$$

in which, $R_{\max}$ and $R_{\min}$ are respectively radius of two ends of cone, γ is the conical angle. b in figure 1 is the axial length of the egg.

According to the membrane theory, the critical stress of the eggshell can be obtained by:

$$\sigma_{\varphi cr}=p_{\varphi cr}\overline{R}\ /h\ . \tag{3}$$

Because γ is very small, $\cos\gamma$ approximates to 1. Like avian eggshells, dinosaur eggshells are mainly composed of calcitic crystallites and a small amount of organic matrix. The structure of dinosaur eggshells available in this paper is very similar to that of avian eggshells. Though E and σ_b of fresh dinosaur eggshells are unknown, they can be referred to that of avian eggshells.

Here we take $\mu=0.25$, the same as avian eggshells. $p_{\varphi cr}$ and $\sigma_{\Phi cr}$ of dinosaur eggs available in this paper are shown in table 2.

According to statistics, σ_b of avian eggshell equals to $0.025E$. As shown in table 2, $\sigma_{\varphi cr}$ of type A is $0.0399E$ greater than σ_b, only when $\sigma_{\varphi cr}$ amounts to σ_b will eggshells of this type be broken. As for types B, C, D and E, things are different. Provided the stress amounts to $\sigma_{\varphi cr}$, eggshells will be broken. If dinosaur eggs of types B, C, D and E were buried evenly in sand, only little load would break them.

2. Analysis of mechanical properties of dinosaur eggs buried vertically in sand

If dinosaur eggs were buried vertically in sand, both ends of each egg turned to be spherical shell under distributive external pressure and the middle portion of egg turned to be conical shell under external pressure along the long axis.

(1) critical pressure of both ends of the egg

The critical external pressure p_{xcrs} can be obtained from the following formula:

$$p_{xcr}=K_1 K_2 K_3 E(h / R)^2 \tag{4}$$

in which , h is the shell thickness, R is the radius. K_1 is the theoretic calculation coefficient (Timoshenko & Gere, 1961) and :

$$K_1=2 / \sqrt{3(1-\mu^2)} . \tag{5}$$

K_2, which decreases with the less of h / R, is called deviation coefficient of the shape. It is introduced to account for that the eggshell is not an ideal sphere, and can be gotten from figure 2. K_3 is introduced to consider the possibility of local subsidence on the eggshell due to being subjected to unequal load or not to being subjected to loading, and can be obtained from figure 3. In figure 3:

$$\lambda=[12(1-\mu^2)]^{1/4} (R / h)^{1/2}. \tag{6}$$

δ expresses the level of subsidence:

$$\delta=\Delta / h \tag{7}$$

in which Δ is the maximum value of subsidence of the real eggshell.

As shown in formula (4), p_{xcrs} is in proportion to $(h / R)^2$. The blunt end of egg is easier to be instable than the pointed end. Therefore, we calculated only p_{xcrs} and σ_{xcrs} at the blunt end. Table 3 shows the results. Here we take $\mu=0.25$, $\delta=0.2$,

$$\sigma_{xcrs}=p_{xcrs} R / (2h). \tag{8}$$

(2) critical external pressure and critical stress of the egg's middle portion along the long axis

The critical total load (Weingarten *et al*., 1965) is:

$$P_{xcr}=2\pi K_x h^2\cos^2\gamma E\,. \tag{9}$$

In which:

$$K_x=\frac{1}{\sqrt{3(1-\mu^2)}}=0.54\left[1-\exp\left(-\frac{1}{16}\sqrt{\frac{\overline{R}}{h}}\right)\right]+0.9\left(\frac{\overline{R}}{L}\right)^2\left(\frac{h}{\overline{R}}\right). \tag{10}$$

The relevant external pressure is:

$$p_{xcr}=2K_x(h/\overline{R})^2\cos^2\gamma E \tag{11}$$

and the stress is:

$$\sigma_{xcr}=P_{xcr}/(2\pi\overline{R}h)=K_x h E\cos^2\gamma/\overline{R}\,. \tag{12}$$

The results are shown in table 4.

As indicated in tables 2, 3 and 4, dinosaur eggs such as types B, C, D and E had little capacity to resist instability. Only by putting them vertically in sand could they bear greater external pressure.

3. Analysis of mechanical properties of dinosaur eggs buried obliquely in sand

If dinosaur eggs were buried obliquely in sand, the long axis forms angle β with the ground. The axial pressure is:

$$p_A=p\sin\beta \tag{13}$$

The lateral pressure is:

$$p_B=p\cos\beta \tag{14}$$

According to the membrane theory, the axial stress produced by p_A is:

$$\sigma_x=p_A R/(2h)=pR\sin\beta/(2h)\,. \tag{15}$$

The lateral stress produced by p_B is:

$$\sigma_\Phi=p_B\overline{R}/h=p\overline{R}\cos\beta/h\,. \tag{16}$$

The maximal axial (critical) stress under p_A is:

$$\sigma_{xcr}=p_{xcr}\overline{R}/(2h)=K_x hE\cos^2\gamma/\overline{R} \tag{17}$$

The maximal lateral (critical) stress under p_B is:

$$\sigma_{\varphi cr}=\frac{KE}{1-\mu^2}\left(\frac{h}{\overline{R}}\right)\cos\gamma\,. \tag{18}$$

According to the breaking criterion of shell buckling when

$$(\sigma_x/\sigma_{xcr})^2+(\sigma_\varphi/\sigma_{\varphi cr})^2=1, \tag{19}$$

the shell would be instable and broken. Substituting (15) – (18) into (19), taking $\cos\gamma=1$, we can get

$$\frac{P_{cr}}{E}=\frac{1}{(G\sin^2\beta+H\cos^2\beta)^{1/2}}\,. \tag{20}$$

Where,

$$G=\frac{1}{4K_x^2}\left(\frac{R}{h}\right)^4, \quad H=\frac{(1-\mu^2)^2}{K^2}\left(\frac{R}{h}\right)^4 \tag{21}$$

Table 5 shows the critical stress when dinosaur eggs were buried in sand with different angle β. Figure 4 indicates variation in the critical pressure with the angle β in four types of dinosaur eggshells.

Discussion

From the foregoing results, we can see that the type A represented by *Ovaloolithus* has great compressive strength, because its σ_{cr} was always greater than σ_b. No matter what angle these eggs were lying in nests with, they were not easy to be broken. This is consistent with the previous found (Young, 1965). It also suggests that dinosaurs represented by *Ovaloolithus* could lay eggs in nests irregularly.

However, types B, C, D and E such as *Macroolithus*, *Elongatoolithus*, *Nanhsiungoolithus* and *Prismatoolithus* do not have enough strength. When they were laid evenly in sand, if σ_{cr} amounted to 1 /4 or 1 /5 of σ_b, the middle portion of egg might be broken by external pressure: If they were buried obliquely in sand, the ability to resist instability in the middle portion of egg would be sharpened. When $\beta<45°$, p_{cr} increases little. When $\beta=60°$, p_{cr} approximates twice as much as when $\beta=0°$. When $\beta=75°$, p_{cr} is about three times. If these eggs were buried vertically in sand (i. e. $\beta=90°$), the ability to resist instability of two ends of the egg would be roughly the same as that of the middle portion. σ_{cr} would approximate σ_b. Hence, those dinosaur eggs with low strength must be buried in sand with the long axis forming certain angle with the ground, their ability to resist the external pressure would be sharpened and the chance of being broken by external pressure would be lowered.

Hypsilophodontid eggs were buried vertically or obliquely in sand (Horner, 1984; Zhao and Li, 1993). Based on the foregoing analysis, hypsilophodontid eggs represented by type E would resist the maximum load when β equaled 70° – 90°. Regarded this as criterion, when $\beta=45° - 75°$, types B, C and D represented by Elongatoolithidae would have the same ability to resist instability as hypsilophodontid eggs buried with 75° – 90° angle.

The dinosaur eggs represented by types B, C and D were found to be arranged regularly in nests (Zhen and Wang, 1963; Young, 1965; Zhao, 1975). The present result shows no difference with this. Also it is reasonable to believe that dinosaur eggshells of types B, C and D represented by Elongatoolithidae and of Hyposilophodontidae (type E) do not have enough strength, and could not provided valid protect. The effective way to solve this problem was to arrange these eggs with certain angle with the ground when they were laid. Therefore, in order to protect the developing embryoes efficiently during incubation, dinosaurs represented by these eggs must adapt their reproductive behaviors. It was necessary and reasonable for dinosaurs to lay eggs regularly in nests. And this suggests that dinosaurs might be more clever than what people have been thinking of them.

图版 I 说明 (Explanations of plate I)

1. 粗皮巨形蛋(*Macroolithus rugustus*)一窝(引自杨钟健，1965，图版 V)。蛋化石作圆形放射状排列，重叠到三层，由外层至内层蛋的倾斜度大约为 40°—70°

   A clutch of *Macroolithus rugustus* from Nanxiong Basin, Guangdong Province; the eggs are arranging in three layers in a circular manner, the inclination of the eggs at the outer layer is about 40 degrees, while those of inner circle are more than 70 degrees

2. 棱齿龙蛋 *Prismatoolithus gebiensis* 一窝(引自赵资奎，李荣，1993，图版 I)。蛋化石直立排列于蛋窝中

   A clutch of hypsilophodontid eggs, *Prismatoolithus gebiensis*, from Bayan Manduhu, Nei Mongol; the eggs stand up vertically in the nest


1


2